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WIND FARM LAYOUT OPTIMIZATION IN COMPLEX TERRAINS USING COMPUTATIONAL FLUID DYNAMICS

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ABSTRACT

The aim of wind farm design is to maximize energy production and minimize cost. In particular, optimizing the placement of turbines in a wind farm is crucial to minimize the wake effects that impact energy production. Most work on wind farm layout optimization has focused on flat terrains and spatially uniform wind regimes. In complex terrains, however, the lack of accurate analytical wake models makes it difficult to evaluate the performance of layouts quickly and accurately as needed for optimization purposes. This paper proposes an algorithm that couples computational fluid dynamics (CFD) with mixed-integer programming (MIP) to optimize layouts in complex terrains. High-fidelity CFD simulations of wake propagation are utilized in the proposed algorithm to constantly improve the accuracy of the predicted wake effects from upstream turbines in complex terrains. By exploiting the deterministic nature of MIP layout solutions, the number of expensive CFD simulations can be reduced significantly. The proposed algorithm is demonstrated on the layout design of a wind farm domain in Carleton-sur-Mer, Quebec, Canada. Results show that the algorithm is capable of producing good wind farm layouts in complex terrains while minimizing the number of computationally expensive wake simulations.

INTRODUCTION

Wind energy is one of the fastest growing sustainable sources of electricity, experiencing substantial growth in recent years [1] due to its relatively high return on investment [2].

The main objective of a wind farm is to maximize annual energy production while minimizing costs. Power production of a wind turbine is dependent on incoming wind speeds, which in turn are dependent on atmospheric conditions, terrain topography, and interference from upstream wind turbines. Hence, any design of onshore wind farms in complex terrains must carefully account for the wake and terrain effects. This paper proposes an algorithm suitable for solving the wind farm layout problem of maximizing energy output in complex terrains.

Most of the work in the literature has focused on wind farm layout optimization (WFLO) problems on flat and uniform topography [1, 3–5]. However, wind speed over complex terrains is very different from that over flat terrains as complex flow structures can form as wind flows over a hilly land. These differences in turn affect the power production of turbines located in the area. The lack of analytical wake models for complex terrains makes it difficult to evaluate and optimize wind farm layouts. Feng and Shen [6] used an adapted Jensen wake model and random search algorithm to generate wind farm layouts in a simple 2D Gaussian hill that is shown to outperform expert guess layouts. The virtual particle model developed by Song et al. [7] is a relatively low-cost wake simulation tool that accounts for the interactions between wake and terrain, thus describes the wake in complex terrains more accurately than the adapted Jensen wake model. However, reducing the number of wake evaluations, and in turn the computational cost, during the optimization process remains a challenge. Hence, the focus of subsequent work [8–10] has been on the coupling between wake modelling and optimization algorithms.

Deterministic approaches such as mixed-integer programming (MIP) models [1, 3, 11, 12] have shown to be promising in solving WFLO problems, as they provide global solutions quickly and can provide optimality bounds for relatively small problems. Furthermore, the deterministic nature of MIP solvers is ideal for drawing comparisons between different problem formulations [11, 12]. In a MIP formulation, the wake interactions are calculated in advance so that optimal layouts can be found efficiently and consistently using algorithms such as branch and bound [3, 11, 13–15]. This deterministic nature of the optimization process is essential for the proposed algorithm, as will described later.

Detailed computational fluid dynamics (CFD) models have been used to simulate complex turbine wake structures and their interactions with the ground [16–20]. Despite the introduction of these models and their obvious necessity in complex terrains, they remain too computationally expensive for general layout optimization. Therefore, CFD simulations should be carried out sparingly in the optimization process. In this approach, the deterministic nature of MIP solvers is crucial to reduce the computational cost.

The objective of this paper is to introduce an algorithm suitable for deterministic optimization techniques to optimize wind farm layout in complex terrains. The proposed algorithm will determine the most promising turbine locations where detailed CFD simulations should be conducted, then integrate the flow field results into a MIP formulation in order to improve the accuracy of layout optimization. In this realistic study, the terrain found at the Carleton Wind Farm in Quebec is used to perform layout optimization. The paper is organized as follows, MIP optimization model and wake modelling, followed by the proposed methodology, and results and discussion of the case study.

OPTIMIZATION MODEL

Mixed integer programming formulations have been developed to solve discrete-variable formulations of the WFLO problem. A MIP consists of an objective function, constraints, and a mix of integer and continuous variables. The WFLO problem can be formulated into a MIP model, where the available land is divided into a number of cells where the turbines can be placed. The proposed formulation, similar to that of the work of Kuo et al. [12], will be to maximize the sum of the kinetic energy per mass of air experienced by each turbine. It is important to note that in a complex terrain, U_0 is a function of location, thus the first U_0^2 term cannot be neglected as with flat terrain WFLO formulations. The detail discussion of the formulation is as follows. Let the wind farm domain be divided into a total of N cells, let K be the number of turbines to be placed (considered a constant in the formulation), and let x_i be a binary variable denoting whether a turbine is placed in the i-th cell, then the optimization problem is

$$\max \sum_{i=1}^{N} \sum_{d\in L} p_d x_i \left[U_0^2 - \sum_{j\in J} (U_0^2 - u_{ij}^2) x_j \right] \tag{1a}$$

$$\text{s.t} \quad \sum_{i=1}^{N} x_i = K \tag{1b}$$

$$d_{ij} x_i + d_{ji} x_j \leq 1.5 \qquad \forall i, j \tag{1c}$$

$$x_i \in \{0,1\} \qquad \forall i = 1, ..., N \tag{1d}$$

where the binary integer terms d_{ij} and d_{ji} indicate the violation of the distance constraint between i-th and j-th cells ($d_{ij} = d_{ji} = 1$ if the distance constraint is violated, $d_{ij} = d_{ji} = 0$ if not), which need to be calculated in advance. In Eq.(1), p_d is the probability of wind state d, and L is the total number of wind states, where a wind state is defined as a (speed, direction) pair. Most importantly, $U_0^2 - u_{ij}^2$ denotes the kinetic energy deficit at cell i caused by turbine at cell j, which is dependent on the wind state. This relationship is illustrated in Fig. 1.

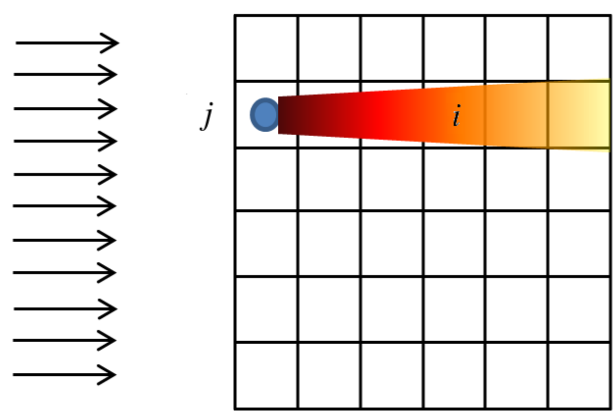


FIGURE 1: Turbine wake created by west wind.

The effects of multiple wakes on wind speed is approximated by using the energy balance approach by Kuo et al. as it offers a form suitable for MIP formulation compared to other wake interaction models (e.g. sum of squares) and also a sound physical basis for applying in complex terrains. The interaction model can be derived by conducting energy balance along a streamtube from the free stream and mixing into the wake, assuming that mixing losses are additive when multiple wakes overlap. In this MIP formulation, the kinetic energy deficit per mass of air caused by a turbine at one location on all remaining locations must be calculated in advance for all possible locations and wind states. Specifically, having a turbine at cell j affect the kinetic energy deficit at all the remaining cells i, and should be calculated for all cells for all wind states. Hence, the number of cell locations multiplied by the number of wind states establishes the number of wake calculations required to use the MIP formulation of the problem. It is not practical to conduct CFD simulations for each of these wake calculations. The proposed approach will enable a wind farm designer to only conduct CFD simulations at a fraction of the total number of wake simulations, by relying on a combination of simplified wake calculations and detailed, full-scale wake simulations.

WAKE MODELLING

One of the most widely used wake models in WFLO literature is the Jensen model [21]. It assumes a linearly expanding wake diameter and uniform velocity profile within the wake. As a result of momentum conservation, the velocity deficit decreases asymptotically with the distance behind the rotor [21]. However, it does not account for terrain effects on the wake.

Rather than using the Jensen model, numerical results from CFD simulations will be used as input for optimization. Each turbine is modelled using the well-established actuator disc model [17,22,23] and the extended $k-\varepsilon$ turbulence model by El Kasmi and Masson [22], as it has been documented that the standard $k-\varepsilon$ model underestimates the velocity deficit [17,22].

Initially, a turbine in flat terrain is simulated and the numerical results used as an initial estimation of the wake effects in complex terrains by using Eq.(2) and Eq.(3). The assumptions made here are that the wake propagates downstream along the terrain surface and that the wake will experience a speed-up factor due to terrain effects similar to that of the wind speed without the presence of a turbine, i.e.

$$u_t(x,y) = S(x,y)u_f, \tag{2}$$

$$u_t^w(x,y) = S(x,y)u_f^w, \tag{3}$$

where u_t and u_f are the free stream wind speeds in complex and flat terrains respectively, $S(x,y)$ is the speed-up factor, and u_t^w and u_f^w describe the wind speed in the wake in complex and flat terrains, respectively. In other words, the speed-up factor due to terrain effects is calculated without the presence of turbines, and then used to "carry" the wakes downstream. The main assumption being that a turbine wake can be superposed into a flow field without turbines, similar implementation was used by Feng and Shen [6] and in several commercial software packages [6]. However, in this work, we relax this assumption and improve the accuracy of the speed-up factor by conducting turbine wake simulations at promising locations.

The interactions between the wind and the rotors after multiple turbines increase the problem difficulty. In the MIP formulation, the multiple wakes are merged using the wake interaction model proposed by Kuo et al. [12], in which energy balance is formulated on overlapping wakes to quantify the energy deficit.

OPTIMIZATION ALGORITHM

In previous sections, the optimization and wake modelling was described individually, the challenge lies in merging the two together since full scale CFD simulations are expensive. In the MIP model, the effects of a single turbine placed in all possible locations are calculated in advance. The interaction effects are then summed up linearly to account for the multiple turbines upstream. As a result, the maximum number of single turbine simulations is the number of possible turbine locations multiplied by the number of wind states. Performing CFD simulations of a single turbine placed in all possible locations can get expensive very quickly as the wind regime becomes more complex and the number of turbine locations increases. Hence, the logic behind the proposed algorithm is that CFD simulations should be conducted only at promising turbine locations. These promising locations can be determined by deterministic optimization methods, so that any change to the solution would arise only from the new information obtained from CFD simulations.

The flowchart of the algorithm is shown in Fig. 2. Firstly, a flow field over the complex terrain without turbines is generated using CFD. In order to account for the terrains effects on turbine wakes, the approximated method (Eq.(2) and Eq.(3)) described in the wake modelling section is used to superpose turbine wake in flat terrain onto complex terrains. In this first iteration, the optimization problem can be solved to determine the initial layout, where the turbines can potentially be placed. At these locations, detailed CFD simulations are performed in order to update the flow field (u_{ij} terms in Eq.(1a)). This process is repeated until no new turbine locations are found. It should be noted that it is not necessary to find the globally optimal solution as long as the optimization algorithm is deterministic such that the solver would converge to a fixed solution in a set criterion, e.g. computational time, in order for the algorithm to terminate. In this case, it is crucial to re-evaluate all the optimal layouts found once the algorithm terminates.

SIMULATION-BASED OPTIMIZATION

Ideally, the u_{ij} terms (Eq.(1a)) should be calculated using high-fidelity CFD simulations. However, this cannot be the case for all possible turbine locations as CFD simulations are computationally expensive. Instead, optimization can be used to narrow down the potential locations where CFD simulations should be

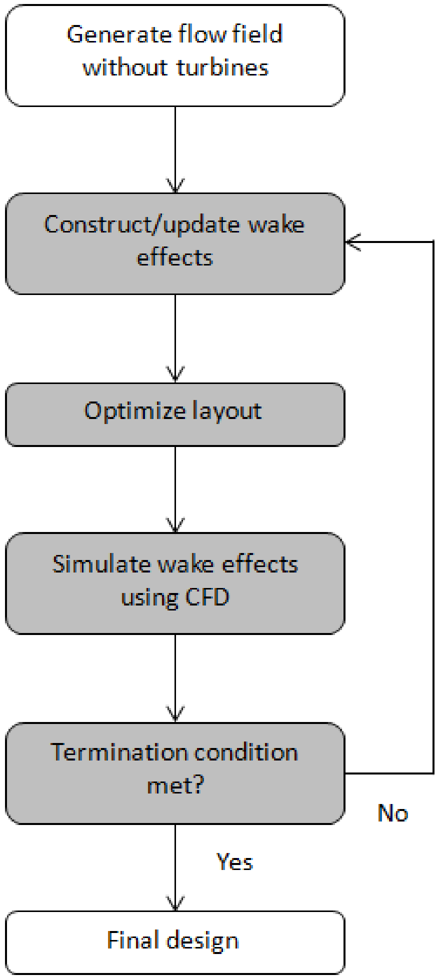


FIGURE 2: Flowchart of the optimization algorithm process

conducted. The underlying assumption is that certain regions in the wind farm domain are more valuable than others and is more likely to have a turbine placed there in an optimal solution.

The commercial CFD code FLUENT is used to compute the flow field. Initially, CFD simulations of the terrain without wind turbines are conducted, for all wind states, to predict wind speeds at hub height to construct $u_t(x,y)$ and $S(x,y)$ from Eq.(2). The $S(x,y)$ term, along with velocity deficits induced by turbine wakes as if they were developing on a flat terrain, will be used in Eq.(3) to construct the approximate flow field in the first iteration. Based on this, the optimization problem is solved to find promising turbine locations. In the first iteration, the number of promising locations equals to the number of turbines. Then, CFD simulations of individual turbines at these locations (for each wind state) are done to improve the accuracy of the flow field, by updating the u_{ij} terms in the MIP model. Once the u_{ij} terms have been updated, the optimization problem can be solved again. If new promising turbine locations have been found, then the CFD simulations are run at these new locations. If no new locations are found, the algorithm will terminate.

CASE STUDY: THE CARLETON-SUR-MER WIND FARM

The topography of a 4 km x 4 km wind farm domain in Carleton-sur-Mer, Quebec, Canada was extracted from Google Maps^{TM} (https://goo.gl/maps/XTpxd), with a surface roughness height assumed to be 2 m. The terrain elevation in meters above sea level is shown in Fig. 3. This wind farm domain is discretized into a uniform 20 x 20 grid of cells, with each cell center separated by a distance of 200 m. The turbines are assumed to have a constant thrust coefficient of 0.88, hub height of 80 m, and rotor diameter of 77 m. The proximity constraint between turbines was set as 2.5 diameters apart, which is passively enforced through the cell grid dimensions.

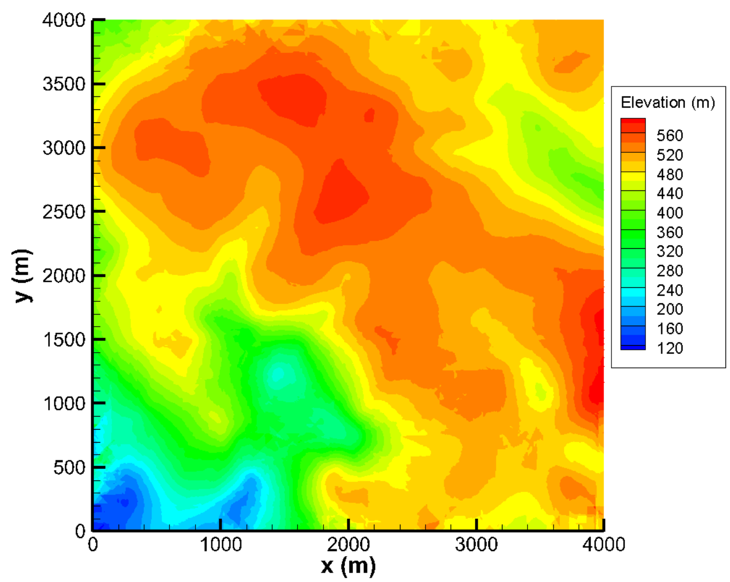


FIGURE 3: Wind farm domain in Carleton-sur-Mer

A power law velocity profile, based on data available from Canadian Wind Energy Atlas [24], is used to describe the wind speed at different altitudes

$$u(y) = 6\left(\frac{y-139}{50}\right)^{0.16}, \quad (4)$$

where y is the height above sea level. This velocity profile is used to define the inlet velocity boundary conditions for CFD simulations. The wind rose used for this domain is shown in Fig. 4, note that the dominant wind direction is the west.

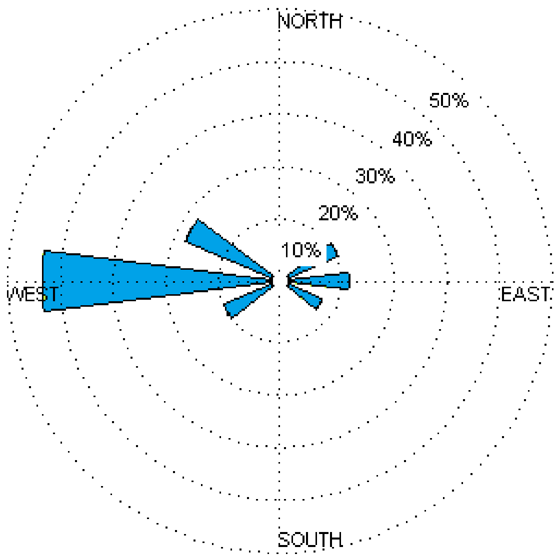


FIGURE 4: Wind rose in Carleton-sur-Mer

RESULTS AND DISCUSSION

The Carleton Wind Farm domain is shown in Fig. 3 with 31 turbines and discretized into a grid of 20 x 20 cells. The wind resource is as described in Fig. 4, with an incoming wind velocity profile as given in Eq.(4). The maximum number of CFD simulations in this scenario is 20 x 20 x 6 = 2400.

The MIP model was implemented using MATLAB and Gurobi 5.6. For the specific wind regime used in this test case, the MIP optimization algorithm converges very quickly, so the bulk of the computational expense comes from the CFD simulations. For each turbine, 6 CFD evaluations are done for the 6 wind directions. Each simulation without turbines contains 1.7 million cells and 2.3 million cells when a turbine is placed in the domain.

In the first iteration, the initial flow field was constructed using CFD simulations of flow over the terrain without turbines. The layout found is shown in Fig. 5a. This approach assumes that these are likely a good set of potential turbine locations, i.e., the implicit assumption is that the wind speed reduction caused by wind turbine wakes is less significant than the speed-up caused by the terrain effects. Hence, it is expected that the majority of the turbines will not move significantly from these locations. The wakes of the turbine located in this layout were simulated in FLUENT and the u_{ij} terms are updated to allow the optimization to more accurately account for the effects of terrains on turbine wakes with detailed CFD data. Once this is done, in the second iteration, the optimized layout becomes Fig. 5b. Note that the turbine located at coordinates (700, 1300) has moved to (900, 3500), so CFD simulations were done to update u_{ij} terms affected by the turbine at (900, 3500). In the third iteration, Fig. 5c, the turbine at (700, 1700) moved to (700, 3500). Another set of CFD simulations with a turbine located at (700, 3500) was done to update the flow field. In the fourth iteration, no turbines moved after the wake effects were updated. The algorithm then terminates and the optimal layout is found. Due to the size of the MIP (i.e. 20 x 20 = 400 binary variables), the optimization solver requires less than 20 seconds to reach optimality in each run. On the other hand, each set of CFD simulations takes approximately 6 hours per turbine. In this specific case, the total run time, including both optimization and CFD simulation, was approximately 190 hours.

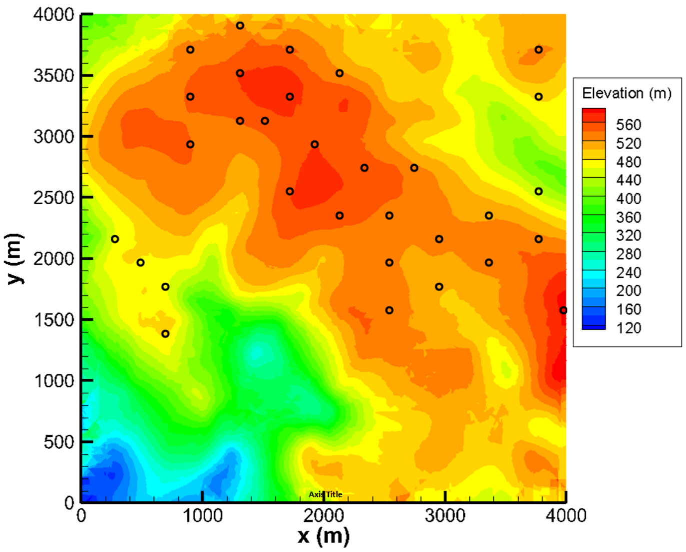
In this example, the number of individual CFD simulations done was 188, much less than the maximum number of 2400, reducing the computational cost significantly. It is expected that such a reduction will still be seen even as the problem size grows.

To assess the performance of the optimal solutions found from each iteration, the progression of the objective values is shown Fig. 6. The effect on wind farm performance by just moving a small number of turbines is clearly demonstrated. In addition, the relatively large change between objective values from iteration 3 and 4 is due to over-predicting of velocity deficit in the flat terrain wake model.

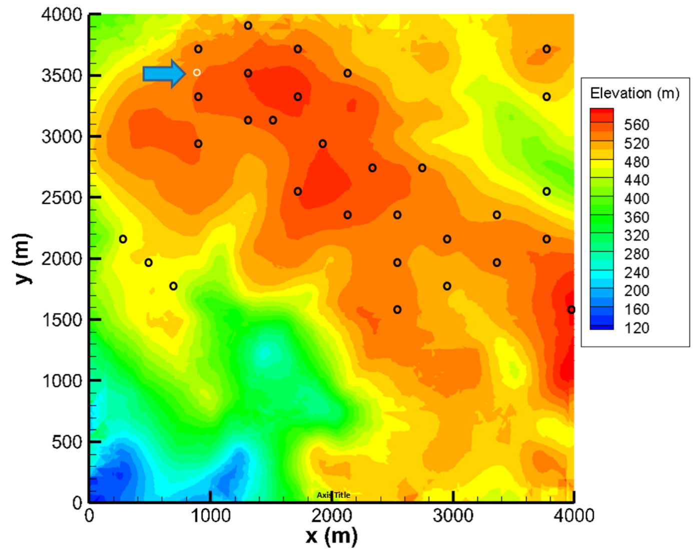
It should be noted that many turbines are placed in higher altitudes as well as regions where local acceleration is experienced. As a reference, if terrain effects are not accounted for in layout optimization, the turbines will space out to minimize the effects of the dominant west wind, shown in Fig. 7. Such a layout would be unable to take advantage of local speed-up effects due to topography.

In some scenarios, it is conceivable that the proposed algorithm may not be able to find the globally optimal solution, as the search may become localized. Due to the complexity of the interactions between the wake and the terrain, it is difficult to predict whether the initial u_{ij} terms would over- or under-predict the actual values. Although there is no formal proof, all the tests indicate that the final solutions found with the proposed algorithm invariably outperform those found when the algorithm is not used, in both solution quality and accuracy.

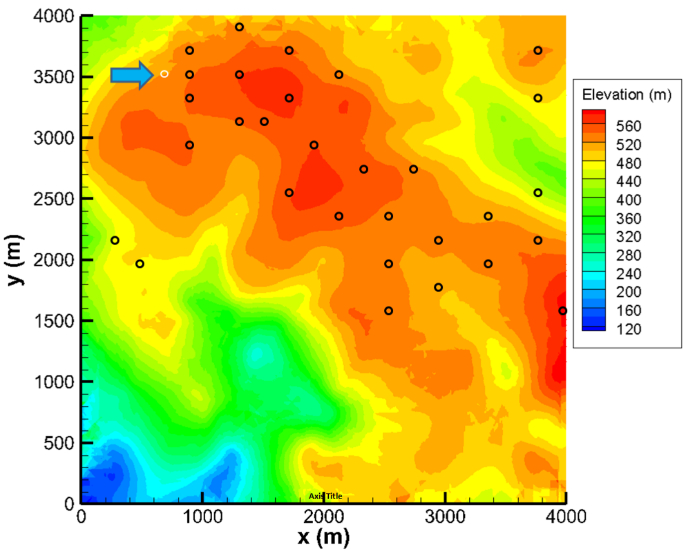
As previously mentioned, the layout found in this scenario using the proposed algorithm may not be the optimal solution if all 2400 CFD simulations of the wake behaviours are available for MIP optimization. Hence, there is a trade-off between computational cost and solution quality. In order to balance between cost and quality, a relaxation factor could be introduced to under- or over-predict the velocity deficit where CFD wake results are not available yet, to allow more possible turbine locations to be explored. This could improve layout solution at the expense of computational cost. This strategy is not implemented



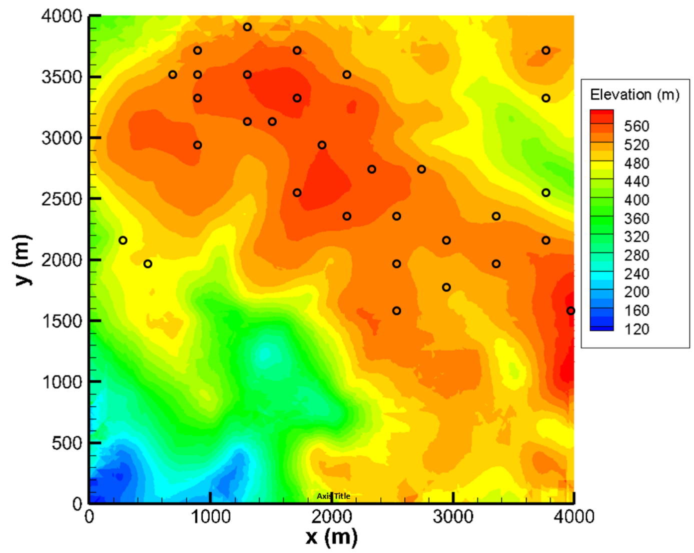
(a) First iteration



(b) Second iteration



(c) Third iteration



(d) Fourth iteration

FIGURE 5: Optimized layout found at the end of each iteration. White circles represent turbines that had moved in that iteration. Note that after only 4 iterations, the algorithm did not identify additional turbine movements that would lead to improvements in the optimization objective.

in this work and will be explored in future studies. On the other hand, calculating the initial wake effects with more robust and accurate wake models, such as virtual particle model [7], would also improve the quality of the initial layout and reduce the number of CFD simulations required to reach an optimal solution.

CONCLUDING REMARKS

An algorithm to optimize wind farm layouts in complex terrains was introduced. This study set out to determine how CFD results can be integrated with layout optimization techniques. This investigation focused on how to implement an algorithm that continuously updates the flow field with detailed simulation data. This is the first WFLO study that combines detailed CFD wake simulations with mathematical programming methods. The

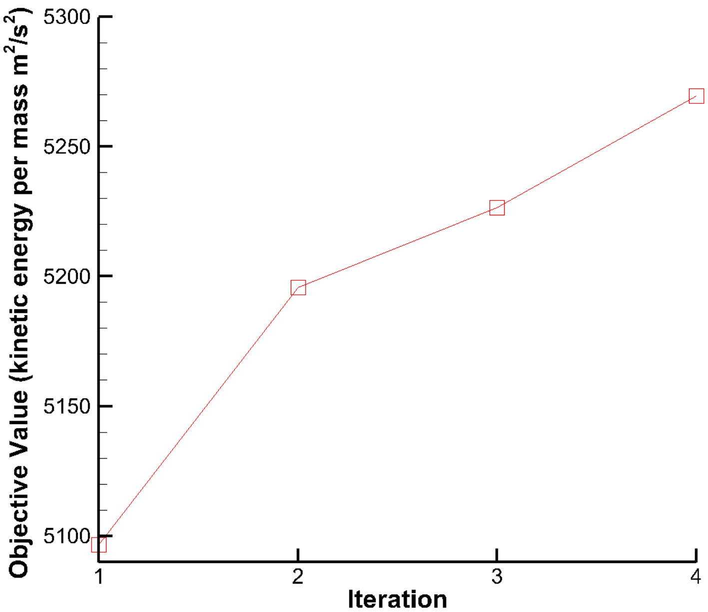


FIGURE 6: Progression of objective value (total kinetic energy per mass of air) with changing number of iterations.

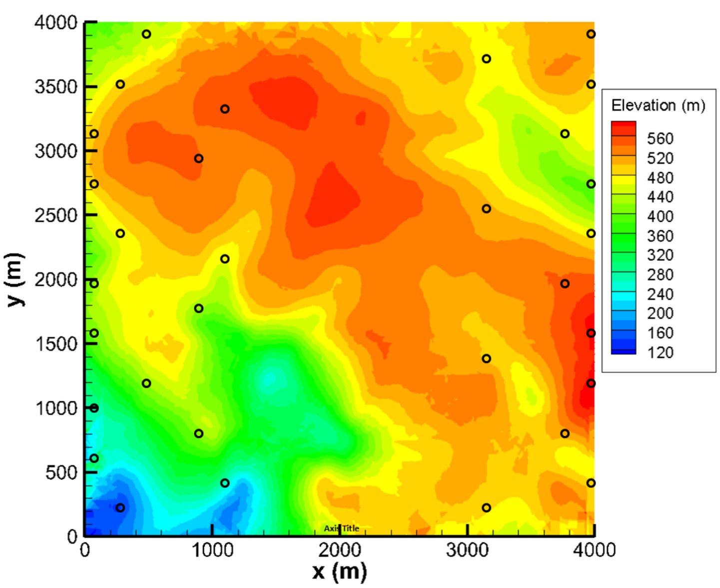


FIGURE 7: Placement of 31 turbines in flat terrain.

results showed that CFD simulations can improve layout optimization by correcting flow field without much additional computational cost.

Considerably more work will need to be done to study the scalability of the approach to larger domains, with more turbines and finer terrain discretizations. However, at this point we are confident that the approach scales well, in fact, our case study is already larger than most test cases used in the WFLO literature. The implication of our findings is that CFD can be a powerful tool in solving the WFLO problem in complex terrains. By using an optimization model to find potential turbine locations with higher probability of being optimal, the computational effort can be significantly reduced.

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