Supplement for LM-cut and Operator Counting Heuristics for Numeric Planning with Simple Conditions: Counter Examples and Additional Proofs

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Translating SCT into RT

Given a SCT $\Pi^{\text{num}} = \langle \mathcal{F}_p, \mathcal{N}, \mathcal{A}, s_I, G \rangle$, we define a transformed task $\Pi^{\text{RT}} = \langle \mathcal{F}_p, \mathcal{N}^{\text{RT}}, \mathcal{A}^{\text{RT}}, s_I^{\text{RT}}, G^{\text{RT}} \rangle$ built as follows. For every numeric expression mentioned in every numeric condition $\psi \in \bar{\mathcal{F}}_n$, we add an additional numeric variable $v^\psi \in \mathcal{N}^{\text{RT}}$, with $s_I[v^\psi] = \sum_{v \in \mathcal{N}} w_v^\psi s_I[v]$. Each numeric condition is replaced by $v^\psi \geq w_0^\psi$ and, for every action a, a numeric effect on every variable v^ψ must be added, with the form $v^\psi \leftarrow v^\psi + \sum_{v \in \mathcal{N}} w_v^\psi k_v^a$, where $v \mathrel{+}= k_v^a \in \mathbb{Q}$ are the original numeric effects of the action. This translation is polynomial in the number of active numeric conditions of the planning task.

Proofs

Proposition 1. *Given a planning task Π, cost partitioning $\{\Pi_i\}_{i=1}^n$, and an admissible heuristic function h_i for each Π_i, the heuristic function $h(s) = \sum_{i=1}^n h_i(s)$ is admissible.*

Proof. Consider the case $n = 2$. For any state s, Let π be an optimal s-plan with the cost $h^*(s)$ for Π. Since Π_1 and Π_2 are the same as Π except for the cost functions, π is also a valid s-plan for both Π_1 and Π_2. Let the cost of π in Π_1 and Π_2 be $h_1'(s)$ and $h_2'(s)$, respectively. Since $\forall a \in \mathcal{A} : \mathsf{cost}_1(a) + \mathsf{cost}_2(a) = \mathsf{cost}(a)$, it holds that $h_1'(s) + h_2'(s) = h^*(s)$. Let $h_1^*(s)$ and $h_2^*(s)$ be the optimal solution costs for Π_1 and Π_2. As h_1 and h_2 are admissible in Π_1 and Π_2, $h_1(s) \leq h_1^*(s) \leq h_1'(s)$ and $h_2(s) \leq h_2^*(s) \leq h_2'(s)$. Thus, $h_1(s) + h_2(s) \leq h_1'(s) + h_2'(s) = h^*(s)$, so $h_1(s) + h_2(s)$ is admissible in Π.

The $n > 2$ case follows by induction. □

The above proof does not use any property which is only included in classical planning but not in numeric planning. Therefore, the proposition of the cost-partitioning also holds for numeric planning.

Proposition 2. *Given an RT and a state s, the problem of computing $h^{\max}$ is NP-hard.*

Proof. We show this result by reduction from the minimization version of the unbounded knapsack problem (mUKP), which is proved to be NP-hard (Zukerman et al. 2001). Let J be a set of elements, each element $i \in J$ has a value w_i and cost c_i. The mUKP problem consists in finding how many times $x_i \in \mathbb{N}^0$ to select each element i to minimize $\sum_{i \in J} c_i x_i$ such that $\sum_{i \in J} w_i x_i \geq W$, where w_i, c_i, W are positive rational numbers. Let $V(X)$ be the optimal cost for an mUKP with the constraint $\sum_{i \in J} w_i x_i \geq X$. Since $V(0) = 0$, we assume that $V(X) = 0$ if $X \leq 0$. Then, $V(X)$ satisfies the following recursion formula:

$$V(X) = \min_{i \in J} V(X - w_i) + c_i.$$

Since $\forall i \in J, w_i > 0$, $X - w_i < X$. Applying the formula recursively, we reaches $X \leq 0$ in a finite number of steps, and the recursion terminates. $V(W)$, the optimal cost for the mUKP, is uniquely determined by the recursion.

Given an instance of the mUKP, we build an RT with no propositional variables and with one numeric variable v. For every element i we have an action a_i such that $\mathsf{pre}(a_i) = \emptyset$, and $\mathsf{num}(a_i) = \{v \mathrel{+}= w_i\}$, $\mathsf{cost}(a_i) = c_i$. We set $s_I[v] = 0$ and G contains one condition $v \geq W$. This RT instance is obtained in $O(|J|)$ time. We show that the solution of the mUKP is equivalent to the solution of $h^{\max}$. By definition,

$$h^{\max}(s_I) = \hat{h}(s_I, G) = \hat{h}(s_I, \{v \geq W\}) = \hat{h}(s_I, v \geq W).$$

For a rational number X, $\hat{h}(s_I, v \geq X) = 0$ if $X \leq 0$ otherwise

$$\hat{h}(s_I, v \geq X) = \min_{a_i \in \mathsf{supp}(v \geq X)} \hat{h}(s, v \geq X - w_i) + \mathsf{cost}(a_i)$$

because all actions have no precondition. Since $\mathsf{num}(a_i) = \{v \mathrel{+}= w_i\}$ for each action a_i and $\mathsf{supp}(v \geq X) = \{a_i \mid i \in J\}$,

$$\hat{h}(s_I, v \geq X) = \min_{i \in J} \hat{h}(s, v \geq X - w_i) + c_i.$$

This is equivalent to $V(W)$, so $h^{\max}(s_I) = \hat{h}(s_I, v \geq W)$ is the optimal cost for mUKP. □

Here, we show an RT where $h^{\text{LM-cut}}_{\text{hbd}}$ returns an inadmissible heuristic value.

Example 3. *Let $\Pi^{\text{RT}} = \langle \mathcal{F}_p, \mathcal{N}, \mathcal{A}, s_I, G \rangle$ be an RT with $\mathcal{F}_p = \{p_1, p_2, p_3, p_4, p_5, g_1, g_2\}$ and $\mathcal{N} = \{v\}$. Let $s_I = \{v = 0\}$, $G = \{g_1, g_2\}$, and $\mathcal{A} = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}\}$, where*

action	pre	eff	cost
a_1	$\emptyset$	p_1	3
a_2	$\emptyset$	p_2	1
a_3	$\emptyset$	p_3	5
a_4	p_1	p_4	1
a_5	p_2	p_5	1
a_6	p_3	$v \mathrel{+}= 1$	0
a_7	p_4, p_5	$v \mathrel{+}= 1$	1
a_8	p_4	g_1	0
a_9	p_5	g_2	0
a_{10}	$v \geq 1$	g_1, g_2	0

We show a hypergraph representation of the task in Figure 3. The optimal plan is $\langle a_3, a_6, a_{10} \rangle$ with the cost of 5.

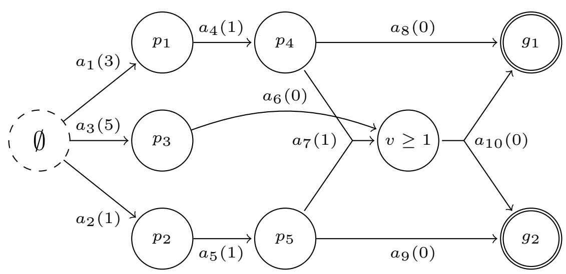


Figure 3: A hypergraph representation of Ex. 3. The action costs are shown in parentheses.

Consider $h^{\text{LM-cut}}_{hbd}$ for this task using the justification graphs for $h^{\max}_{hbd}$. Let Π^{RT}_i be the planning task with reduced action costs cost_i after extracting the i th cut L_i in the computation of $h^{\text{LM-cut}}_{hbd}$. For each $\psi \in \mathcal{F}_p \cup \bar{\mathcal{F}}_n$, $h^{\max}_{hbd}(s_I, \psi)$ in each task is as follows:

$h^{\max}_{\text{hbd}}(s_I, \psi)$	Π^{RT}	Π^{RT}_1	Π^{RT}_2	Π^{RT}_3	Π^{RT}_4
$\emptyset$	0	0	0	0	0
p_1	3	3	0	0	0
p_2	1	1	1	1	0
p_3	5	5	5	5	5
p_4	4	3	0	0	0
p_5	2	2	2	1	0
$v \geq 1$	4	3	2	1	0
g_1	4	3	0	0	0
g_2	2	2	2	1	0

In Π^{RT} and Π^{RT}_1,

$$\mathsf{pcf}_{hbd}(s_I, a_6, v \geq 1) = \mathsf{pcf}_{hbd}(s_I, a_7, v \geq 1) = p_4$$

and $p_4 \in N^g$ since

$$\begin{aligned} \operatorname*{argmin}_{a \in \mathsf{supp}(v \geq 1)} h^{\max}_{hbd}(s, \mathsf{pre}(a)) &= \operatorname*{argmin}_{a \in \{a_6, a_7\}} h^{\max}_{hbd}(s, \mathsf{pre}(a)) \\ &= \{a_7\}, \end{aligned}$$

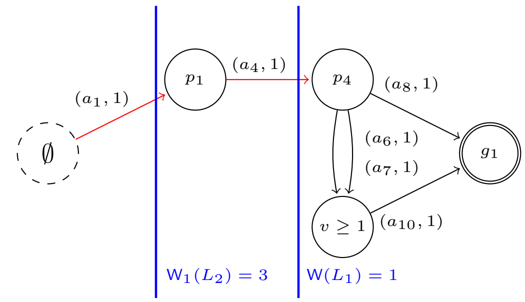
$$\operatorname*{argmax}_{\psi \in \mathsf{supp}(a_7)} h^{\max}_{hbd}(s, \psi) = \operatorname*{argmax}_{\psi \in \{p_4, p_5\}} h^{\max}_{hbd}(s, \psi) = \{p_4\},$$

and $\mathsf{cost}(a_7) = 0$. Likewise,

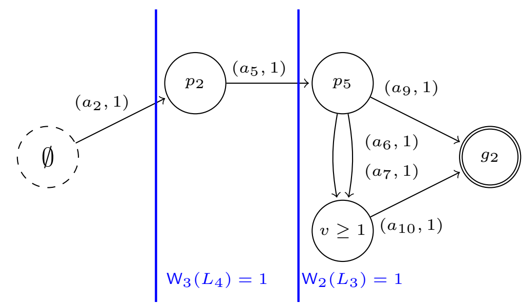
$$\mathsf{pcf}_{hbd}(s_I, a_6, v \geq 1) = \mathsf{pcf}_{hbd}(s_I, a_7, v \geq 1) = p_5$$

and $p_5 \in N^g$ in Π^{RT}_2 and Π^{RT}_3. The justification graphs are shown in Figure 3. Note that p_3 never appears in the justification graphs because $\mathsf{pcf}_{hbd}(s_I, a_6, v \geq 1) \neq p_3$. We have

$$h^{\max}_{hbd}(s_I) = 4 < h^*(s_I) = 5 < h^{\text{LM-cut}}_c(s_I) = 6.$$



(a) The first and second cuts.



(b) The third and fourth cuts.

Figure 4: JGs for the RT in Ex. 3. The functions W, W_1, W_2, and W_3 denote the cut weights of the LM-cut procedures, where action costs are reduced in each iteration.

The inadmissibility of $h^{\text{LM-cut}}_{\text{hbd}}$ follows directly from Ex. 3.

Proposition 3. *The LM-cut heuristic based on the pcf_{hbd} JG, $h^{\text{LM-cut}}_{hbd}$, is inadmissible*

Lemma 1. *Assume an RT of a non-zero cost. Let $\mathcal{G}$ be the JG corresponding to Π^{RT}, and let L be a directed cut in $\mathcal{G}$ that separates $n_\emptyset$ from n_g, such that $\mathsf{W}(L) > 0$. Then,*

1. *$\partial^{\mathsf{in}}(L)$ is a disjunctive fact landmark.*
2. *$\mathsf{label}(L)$ is a disjunctive action landmark.*

Proof. Let π be a plan for Π^{RT}. Let us construct the subsequence π' of the plan π. Let a_g be the first action in π that achieves the atom g. Note that by construction $s \not\models g$, thus such action should exist. For the action a_g we chose the first action in π that achieves $\mathsf{pcf}(a_g)$, and repeat the process until we reach a fact ψ such that $s \models \psi$. Note that by construction π' induces a path from $n_\emptyset$ to n_g in the JG. Thus, for every cut L that separates $n_\emptyset$ from n_g we have that at least one fact in $\partial^{\mathsf{in}}(L)$ is achieved by π', and $\pi' \cap \mathsf{label}(L) \neq \emptyset$. Thus, $\partial^{\mathsf{in}}(L)$ is a disjunctive fact landmark. Note that a_0, an artificial action label, is never included in L. If $a_0 \in \mathsf{label}(L)$, $\exists (n_\emptyset, n_\psi; a_0) \in L, n_\psi \in N^g$. Since the cost of a_0 is zero, $n_\emptyset \in N^g$, and this contradicts that

$L = (N^0, N^g \cup N^b)$. Therefore, $\mathsf{label}(L)$ is a disjunctive action landmark. $\square$

We show that the justification graphs based on $\mathsf{pcf}_{\mathsf{cri}}$ and $\mathsf{pcf}_{\mathsf{hbd}}$ justify $h^{\max}_{\mathsf{cri}}$ and $h^{\max}_{\mathsf{hbd}}$, respectively.

Proposition 4. *Given an* RT *a state s, and the justification graph based on pcf_{cri}, the weight of the shortest path from $n_\emptyset$ to $n_\psi \in N$ is equal to $h^{\max}_{cri}(s_I, \psi)$.*

Proof. The shortest paths to other nodes can be incrementally computed in the topological order, and the first node is $n_\emptyset$. Therefore, we assume that the weight $w(n_{\psi'})$ of the shortest path from $n_\emptyset$ to $n_{\psi'}$ is already known for all $(n_{\psi'}, n_\psi; a) \in E$. Assume that $w(n_{\psi'}) = h^{\max}_{\mathsf{cri}}(s, \psi')$ for all $(n_{\psi'}, n_\psi; a) \in E$. This is correct for $\psi' = \emptyset$ because $w(n_\emptyset) = h^{\max}_{\mathsf{cri}}(s, \emptyset) = 0$. Then,

$$\begin{aligned} w(n_\psi) &= \min_{(n_{\psi'}, n_\psi; a) \in E} \mathsf{m}_a(s, \psi)\mathsf{cost}(a) + w(n_{\psi'}) \\ &= \min_{(n_{\psi'}, n_\psi; a) \in E} \mathsf{m}_a(s, \psi)\mathsf{cost}(a) + h^{\max}_{\mathsf{cri}}(s, \psi'). \end{aligned}$$

Since $\psi' = \mathsf{pcf}_{\mathsf{cri}}(s, a) \in \operatorname{argmax}_{\hat{\psi} \in \mathsf{pre}(a)} h^{\max}_{\mathsf{cri}}(s, \hat{\psi})$,

$$h^{\max}_{\mathsf{cri}}(s, \psi') = \max_{\hat{\psi} \in \mathsf{pre}(a)} h^{\max}_{\mathsf{cri}}(s, \hat{\psi}) = h^{\max}_{\mathsf{cri}}(s, \mathsf{pre}(a)).$$

Because $\mathsf{supp}(\psi) = \{a \mid (n_{\psi'}, n_\psi; a) \in E\}$,

$$\begin{aligned} w(n_\psi) &= \min_{a \in \mathsf{supp}(\psi)} \mathsf{m}_a(s, \psi)\mathsf{cost}(a) + h^{\max}_{\mathsf{cri}}(s, \mathsf{pre}(a)) \\ &= h^{\max}_{\mathsf{cri}}(s, \psi). \end{aligned}$$

By mathematical induction, the shortest path from $n_\emptyset$ to any node n_ψ is equal to $h^{\max}_{\mathsf{cri}}(s, \psi)$. $\square$

Proposition 5. *Given an* RT *a state s, and the justification graph based on pcf_{hbd}, the weight of the shortest path from $n_\emptyset$ to $n_\psi \in N$ is equal to $h^{\max}_{hbd}(s_I, \psi)$.*

Proof. The shortest paths to other nodes can be incrementally computed in the topological order, and the first node is $n_\emptyset$. Therefore, we assume that the weight $w(n_{\psi'})$ of the shortest path from $n_\emptyset$ to $n_{\psi'}$ is already known for all $(n_{\psi'}, n_\psi; a) \in E$. Assume that $w(n_{\psi'}) = h^{\max}_{\mathsf{hbd}}(s, \psi')$ for all $(n_{\psi'}, n_\psi; a) \in E$. This is correct for $\psi' = \emptyset$ because $w(n_\emptyset) = h^{\max}_{\mathsf{hbd}}(s, \emptyset) = 0$. Then,

$$\begin{aligned} w(n_\psi) &= \min_{(n_{\psi'}, n_\psi; a) \in E} \mathsf{m}_a(s, \psi)\mathsf{cost}(a) + w(n_{\psi'}) \\ &= \min_{(n_{\psi'}, n_\psi; a) \in E} \mathsf{m}_a(s, \psi)\mathsf{cost}(a) + h^{\max}_{\mathsf{hbd}}(s, \psi'). \end{aligned}$$

If $\psi \in \mathcal{F}_p$, since

$$\psi' = \mathsf{pcf}_{\mathsf{hbd}}(s, a, \psi) \in \operatorname*{argmax}_{\hat{\psi} \in \mathsf{pre}(a)} h^{\max}_{\mathsf{hbd}}(s, \hat{\psi})$$

and

$$\mathsf{supp}(\psi) = \{a \mid (n_{\psi'}, n_\psi; a) \in E\},$$

$$h^{\max}_{\mathsf{hbd}}(s, \psi') = \max_{\hat{\psi} \in \mathsf{pre}(a)} h^{\max}_{\mathsf{hbd}}(s, \hat{\psi}) = h^{\max}_{\mathsf{hbd}}(s, \mathsf{pre}(a)).$$

Thus,

$$\begin{aligned} w(n_\psi) &= \min_{a \in \mathsf{supp}(\psi)} \mathsf{m}_a(s, \psi)\mathsf{cost}(a) + h^{\max}_{\mathsf{hbd}}(s, \mathsf{pre}(a)) \\ &= h^{\max}_{\mathsf{hbd}}(s, \psi). \end{aligned}$$

Now, assume that $\psi \in \bar{\mathcal{F}}_n$.

$$h^{\max}_{\mathsf{hbd}}(s, \psi') = \max_{\hat{\psi} \in \mathsf{pre}(\hat{a})} h^{\max}_{\mathsf{hbd}}(s, \hat{\psi})$$

where

$$\hat{a} = \operatorname*{argmin}_{a' \in \mathsf{supp}(\psi)} h^{\max}_{\mathsf{hbd}}(s, \mathsf{pre}(a')).$$

Since this is independent of ψ' and a,

$$\begin{aligned} w(n_\psi) \quad = \quad & \min\nolimits_{a \in \mathsf{supp}(\psi)} \mathsf{m}_a(s, \psi)\mathsf{cost}(a) \\ & + \max\nolimits_{\hat{\psi} \in \mathsf{pre}(\hat{a})} h^{\max}_{\mathsf{hbd}}(s, \hat{\psi}). \end{aligned}$$

As

$$\begin{aligned} & h^{\max}_{\mathsf{hbd}}(s, \hat{\psi}) \\ & = \min\nolimits_{a' \in \mathsf{supp}(\psi)} \max\nolimits_{\hat{\psi} \in \mathsf{pre}(a')} h^{\max}_{\mathsf{hbd}}(s, \hat{\psi}) \\ & = \min\nolimits_{a' \in \mathsf{supp}(\psi)} h^{\max}_{\mathsf{hbd}}(s, \mathsf{pre}(a')), \end{aligned}$$

$$\begin{aligned} w(n_\psi) \quad = \quad & \min\nolimits_{a \in \mathsf{supp}(\psi)} \mathsf{m}_a(s, \psi)\mathsf{cost}(a) \\ & + \min\nolimits_{a' \in \mathsf{supp}(\psi)} h^{\max}_{\mathsf{hbd}}(s, \mathsf{pre}(a')) \\ = \quad & h^{\max}_{\mathsf{hbd}}(s, \psi) \end{aligned}$$

By mathematical induction, the shortest path from $n_\emptyset$ to any node n_ψ is equal to $h^{\max}_{\mathsf{hbd}}(s, \psi)$. $\square$

References

Zukerman, M.; Jia, L.; Neame, T. D.; and Woeginger, G. J. 2001. A polynomially solvable special case of the unbounded knapsack problem. *Oper. Res. Lett.* 29(1): 13–16.